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Equivalence of Constrained Models

M. Hortaçsu[1] and K. Ülker[1]

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We study two constrained scalar models. While there seems to be equivalence when the partially integrated Feynman path integral is expanded graphically, the dynamical behaviors of the two models are different when quantization is done using Dirac constraint analysis.

1. INTRODUCTION

There are many models in the literature which claim that they are equivalent to quantum chromodynamics (QCD) in some sense. We can cite recent work by Hasenfratz and Hasenfratz(1) or the work of Akdeniz *et al.*(2) among many papers concerning this topic. We mention these two papers since in these papers the starting Lagrangians look very different, although the effective Lagrangians obtained after some manipulations are made are exactly the same. There are many other papers in the same spirit. One example is ref. 3, which, in some sense, gave rise to ref. 1. Another example is ref. 4, which was actually a pioneering paper in this endeavor of finding models whose effective Lagrangian looks like the standard model.

It was always a puzzle to us how similar manipulations made on very different looking Lagrange functions resulted in completely the same effective Lagrangian. In this note we try to investigate this phenomenon using scalar models. We think that our results in the scalar case may give additional information on this phenomenon.

We will use two constrained scalar models. In ref. 1, the authors imposed the constraint $J^f_\mu = \overline{\Psi} i\gamma_\mu\tau_a\psi = 0$ on the fields of the free spinor Lagrangian. For this model to be resembled here we first study the case where we write a Lagrangian which is essentially equivalent to $L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$. We will impose a different constraint, though, since a constraint $\phi\partial_\mu\phi = 0$ results in a truly

[1] Physics Department, Faculty of Science and Letters, I.T.U., 80626 Maslak, Istanbul, Turkey.

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trivial model. We can also introduce inner symmetry to the theory and make an $O(N)$ model along similar lines. For the time being we do not pursue this.

The authors in ref. 2 impose the condition that their current J^f_μ equals the product of vector fields instead of zero, $J^f_\mu = A_\mu A^2$, which differs from the constraint used in ref. 1. This complicates the problem, but all of the additional fields introduced to the model decouple and at the end only one vector field survives. The propagator for this field, and only for this field is generated in the one-loop calculation. At this point the resulting effective theory looks exactly like that of ref. 1.

We have doubts whether these two models are actually equivalent to QCD in all aspects. One may refer to an old work of Wilson[(5)] and to a more recent work of Zinn-Justin,[(6)] and using the calculations made in ref. 7, claim that these two models are actually examples of trivial models.[(8)]

We will not dwell on these points here. We will only investigate in what sense two models are equivalent when the effective Lagrangians derived from them seem so. In the next section we present two constrained scalar models. We get a theory which is totally trivial if we impose the current made out of scalar fields equal to zero, the analogous case as given in Ref. 1. We instead use two models where the current is equal to one and two auxiliary fields, thus introducing 8 and 16 new degrees of freedom, respectively, plus constraints that will eliminate these. We study the Dirac bracket relations satisfied by the respective fields. We see that the new introduced vector fields via the constraint equations somehow replace the canonical momentum of the scalar field.

In Section 3 we derive effective Lagrangians for these two cases and show why do they seem to be equivalent on this level. We end with some remarks.

2. QUANTIZATION OF THE MODELS USING DIRAC CONSTRAINT ANALYSIS

2.1. We start with

$$L_A = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + ig\lambda_\mu\phi\partial^\mu\phi + \tfrac{1}{2}g^2\phi^2\lambda^2 \tag{1}$$

We are in four-dimensional Minkowski space and μ takes the values zero to three. Here λ_μ is an auxiliary field with no kinetic term. g is a coupling constant.

The equations of motion are

$$\partial_\mu\partial^\mu\phi + ig\partial_\mu(\lambda^\mu\phi) = ig\lambda_\mu\partial^\mu\phi + g^2\lambda^2\phi \tag{2}$$

$$ig\phi\partial_\mu\phi = -g^2\lambda_\mu\phi^2 \tag{3}$$

which can be shown to be equivalent to

$$\partial_\mu \partial^\mu \phi = 0 \tag{4}$$

In this calculation we use the methods given in Dirac's book.[9] The canonical momenta are

$$\pi_\phi = (\partial_0 + ig\lambda_0)\phi \tag{5a}$$

and

$$\pi_{\lambda_\mu} = 0 \tag{5b}$$

which gives us four primary constraints.

The canonical Hamiltonian is

$$H = \tfrac{1}{2}(\pi_\phi - ig\lambda_0\phi)^2 + ig\lambda_i\phi\partial_i\phi + \tfrac{1}{2}\partial_i\phi\partial_i\phi - \tfrac{1}{2}g^2\lambda^2\phi^2 \tag{6}$$

We get secondary constraints when we set the Poisson bracket of H with π_{λ_μ} equal to zero,

$$K_\mu = \frac{d}{dt}\pi_{\lambda_\mu} = (ig\phi\pi_\phi + g^2\lambda_0\phi^2)g_{0\mu} - ig\phi\partial_i\phi g_{i\mu} + g^2\lambda_\mu\phi^2 = 0 \tag{7}$$

We take

$$H_E = H + c_\mu \pi_{\lambda_\mu} \tag{8}$$

and further calculate

$$\frac{d}{dt}K_\mu = [H_E, K_\mu] \tag{9}$$

where square brackets mean Poisson brackets. Note that λ_0 appears in K_0, and the λ_i in K_i. We get one equation with c_0 when $[H_E, K_0]$ is calculated, which fixes the value of c_0 and does not give additional constraints on the system. $[H_E, K_i]$ give us equations which fix c_i. We do not get any additional constraints.

We can calculate the Poisson brackets between the different constraints,

$$[\pi_{\lambda_0}, K_0] = 2g^2\phi^2 g_{00}, \qquad [\phi_{\lambda_i}, K_j] = -g^2\phi^2 g_{ij}$$

$$[K_0, K_i] = g^2\phi\partial_i\phi - 2ig^3\lambda_i\phi^2$$

All the other brackets of the constraints with each other are zero. We see that all these brackets are second class. We calculate Dirac brackets between different fields,

$$[\phi(x), \lambda_0(y)]^{\mathrm{D}} = \frac{i}{2g\phi}\delta^3(x-y) \tag{10a}$$

$$[\phi(x), \lambda_i] = 0 \tag{10b}$$

which shows that λ_0 is like π_ϕ, and λ_i decouples. We can set λ_i equal to zero.

2.2. We propose another model where

$$L_B = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + ig(\lambda_\mu + A_\mu)\phi\partial^\mu\phi - gA_\mu\lambda^\mu A^2 \tag{11}$$

The primary constraints are

$$\pi_{A_\mu} = 0 \tag{12}$$

$$\pi_{\lambda_\mu} = 0 \tag{13}$$

The Hamiltonian reads

$$H = \tfrac{1}{2}(\pi_\phi - ig(\lambda_0 + A_0)\phi)^2 + \tfrac{1}{2}\partial_i\phi\partial_i\phi + ig(\lambda_i + A_i)\phi\partial_i\phi + g\lambda_\mu A^\mu A^2 \tag{14}$$

where

$$\pi_\phi = \partial_0\phi + ig(\lambda_0 + A_0)\phi \tag{15}$$

$$H_E = H + c_1^\mu\pi_{A_\mu} + c_2^\mu\pi_{\lambda_\mu} \tag{16}$$

Secondary constraints

$$[H_E, \pi_{A_\mu}] = Q_\mu^1 = 0 \tag{17}$$

$$[H_E, \pi_{\lambda_\mu}] = Q_\mu^2 = 0 \tag{18}$$

are given as

$$Q_\mu^1 = ig\phi(\pi_\phi - ig(\lambda_0 + A_0)\phi)g_{\mu 0} - ig\phi\partial_i\phi g_{i\mu} - g\lambda_\mu A^2 - 2g\lambda_\nu A^\nu A_\mu \tag{19}$$

$$Q_\mu^2 = ig\phi(\pi_\phi - ig(\lambda_0 + A_0)\phi)g_{\mu 0} - ig\phi\partial_i\phi g_{i\mu} - gA_\mu A^2 \tag{20}$$

We see that the system is closed, since the Poisson brackets of Q_μ^1, Q_μ^2 with H_E involve eight coupled equations for c_μ^1 and c_μ^2. We get no further constraints.

When we calculate the Poisson brackets of the constraints with each other, we see that they are all of second class. We have 16 second-class constraints and 18 degrees of freedom. We have traded some of our fields in terms of others, but we did not change the number of independent variables.

Now we can calculate the Dirac brackets between different fields. We are particularly interested in the brackets between ϕ and A_μ, λ_μ, since the Poisson brackets between the same fields are zero. The effect of the constraints in the system are reflected in the Dirac brackets; hence, they do not vanish when they are taken between ϕ and the auxiliary fields. We give below the result of some sample calculations:

$$[\phi(x), A_1(y)]^D \phi(y) = \frac{i\phi^2(-2A_0 A_1)[g\phi^2(A^2 - 2\mathbf{A}^2) - 3A^4]}{\Delta}\delta^{(3)}(x-y) \tag{21a}$$

$$[\phi(x), A_0(y)]^D \phi(y) = \frac{i\phi^2(A^2 - 2\mathbf{A}^2)[g\phi^2(A^2 - 2\mathbf{A}^2) - 3A^4]}{\Delta}\delta^{(3)}(x-y) \tag{21b}$$

$$[\phi(x), \lambda_0(y)]^D \phi(y) = \frac{i\phi^2(A^2 - 2\mathbf{A}^2)[2g\phi^2(A^2 - 2\mathbf{A}^2) - 3A^4 - 4A^2\lambda_\mu A^\mu]}{\Delta}\delta^{(3)}(x-y) \tag{21c}$$

$$[\phi(x), \lambda_2(y)]^D \phi(y) = 2i\frac{\phi^2[-3A_0 A_2(A^4 - 4A^2 A_\mu\lambda^\mu) - 3A^4(A_0\lambda_2 + A_2\lambda_0)]}{\Delta}\delta^{(3)}(x-y) \tag{21d}$$

where

$$\Delta = [g^2\phi^4(A^2 - 2\mathbf{A}^2)^2 - (6A^2 + 4\lambda_\mu A^\mu)(A^2 - 2\mathbf{A}^2)A^2 g\phi^2 + 9A^8] \tag{22}$$

Here $\mathbf{A}^2$ means the three-vector A squared.

Upon quantization we see that A_μ and λ_μ seem to contain part of π_ϕ. We expect only π_ϕ to have nonzero commutation relations with ϕ and in this model both A_μ and λ_μ also will have nonzero commutations with ϕ. The constraints $Q^1_\mu = 0$ and $Q^2_\mu = 0$ relate π_ϕ to these fields.

The model we have studied seems to be considerably different from the one studied in the first section. The fields in the model have nonzero Dirac brackets, so we cannot set them equal to zero, as in the previous model. The space components of the vector fields do not decouple and cannot be set to zero.

Note that in both of these models the degrees of freedom is two. In model A we start with 10 degrees of freedom, 2 for the ϕ and 8 for the λ field and their respective momenta. Eight constraints reduce these to 2. In model B we start with 18 degrees of freedom since we have two vector particles. Sixteen constraint equations reduce this number to 2. As far as the equations of motion are considered, these two models do not seem to be alike.

3. FEYNMAN RULES USING THE PATH INTEGRAL

Here we study the two models using Feynman diagram expansions of the path integral after the integral is partially integrated. We start by studying model A, then contrast our results with that of model B.

3.1. Here the path integral is written as

$$Z = \int d\phi d\pi_\phi d\lambda_\mu d\pi_{\lambda_\mu} \, \delta(\pi_{\lambda_\mu})\delta(K_\mu) \det M_{\mu\nu} \exp iS \tag{23}$$

where

$$M_{\mu\nu} = \frac{\partial K_\mu}{\partial \lambda_\nu} \tag{24a}$$

$$S = \int d^4x[\pi_\phi \partial_0 \phi + \pi_{\lambda_\mu}\partial_0\lambda_\mu - H_E] \tag{24b}$$

We write the Dirac delta functions in the integral form, introducing new variables A_μ, and express the determinant in the exponential form using ghost fields:

$$\delta(K_\mu) = \frac{1}{2\pi}\int dA_\mu e^{-iA_\mu K^\mu}$$

$$\det M_{\mu\nu} = \int dc_\mu^+ \, dc_\nu \, e^{ic_\mu^+ M^{\mu\nu} c_\nu}$$

The integrations over the momenta and ϕ are performed easily and we end up with

$$Z = N \int dA_\mu d\lambda_\nu dc_\alpha^+ \, dc_\beta$$
$$\times \exp\{-\tfrac{1}{2} \operatorname{tr} \log [-\partial_\mu\partial^\mu + igN_\mu\partial^\mu - ig\partial_\mu N^\mu + g^2 (\tfrac{1}{2}\lambda^2 - A_\mu\lambda^\mu$$
$$-\tfrac{1}{2} A_0^2 + c_0^+ \, c_0 + c_\mu^+ c^\mu)]\} \tag{25}$$

where we define $N_\mu = \lambda_\mu - A_\mu$. We can calculate the inverse propagator $D_{\mu\nu}^{-1}$ for the N_μ field by taking two derivatives of Eq. (25) with respect to the N_μ field. In the momentum representation we get

$$D_{\mu\nu}^{-1}(q) = -g^2 \int \frac{d^4p}{(2\pi)^4} \frac{(p_\mu + q_\mu)(p_\nu + 2q_\nu)}{p^2(p+q)^2}$$
$$= -g^2 \frac{\Gamma(\epsilon)}{6(4\pi)^2}(g_{\mu\nu} q^2 - 10 q_\mu q_\nu) \tag{26}$$

which looks like the massless vector boson propagator, at least in a particular gauge. Note that all the components of the vector field have nonzero propagation.

All other fields have zero propagators if we use dimensional regularization. Here we set $\int d^4p(1/p^2) = 0$. When we drop all the fields with zero propagators we end up with

$$S_{\text{eff}} = -\tfrac{1}{2}\,\text{Tr}\,\log(-\partial^2 + igN_\mu\partial^\mu - ig\partial^\mu N_\mu) \tag{27}$$

Upon expanding the logarithm we can evaluate the multipoint functions for the N^μ fields. Equation (26) dictates a necessary condition on the coupling constant g, though, to have a well-defined expression for the propagator function given by this equation, which reads

$$g^2\,\frac{\Gamma(\epsilon)}{6(4\pi)^2} = 1 \tag{28}$$

This condition makes the model asymptotically free in the ultraviolet regime.

By taking all the nonvanishing terms we see that for the composite field λ_μ the effective Lagrangian can be written as

$$\begin{aligned} L_{\text{eff}} = {} & \tfrac{1}{2}\partial_\mu N_\nu\partial^\mu N_\nu + \partial_\mu N_\nu\partial^\nu N^\mu + gf^{\mu\nu\rho}\,N^\mu N^\nu N^\rho \\ & + g^2 V_{\mu\nu\rho\sigma}\,N^\mu\,N^\nu\,N^\rho\,N^\sigma \end{aligned} \tag{29}$$

Here $f_{\mu\nu\rho}$ is proportional to momentum and Kronecker deltas and $V_{\mu\nu\rho\sigma}$ is made out of Kronecker deltas. Higher order functions, starting with the fifth-point function, drop with higher powers of g. For example, the five-point function goes as g^5. They do not fit into this scheme of the effective Lagrangian and are calculated as loop corrections.

Here we calculated the Feynman rules for this model and showed that apart from the restriction dictated by Eq. (28), we get rules similar to those of a gauge theory. One can calculate physical processes using these rules and get free parton model results, as is the case in a similar model,[7] due to the restriction dictated by Eq. (28). All the physical processes that involve interactions will involve powers of the coupling constant, which goes to zero. Any possible divergences due to loops will be canceled by the zeroes coming from extra powers of the coupling constant. Only terms which do not involve any interactions are finite. These terms are the same as those given in the free-field case.

3.2. The path integral for model B in the Hamiltonian formalism is written as

$$\int dA_\mu\,d\pi_{A_\mu}\,d\pi_{\lambda_\mu}\,d\lambda_\mu d\phi\,d\pi_\phi\delta(\pi_{\lambda_\mu})\delta\,(\pi_{A_\mu})\,\delta(Q^1_\nu)\delta\,(Q^2_\nu)(\det M)\exp iS \tag{30}$$

Here

$$S = \int d^4x\, [\pi_\phi \partial_0 \phi + \pi_{A_\mu}\, \partial_0\, A_\mu + \pi_{\lambda_\mu}\, \partial_0 \lambda_\mu - \frac{1}{2} [\pi_\phi + ig(\lambda_0 + A_0)\phi]^2$$

$$- \frac{1}{2}\, \partial_i \phi \partial_i \phi - ig(\lambda_i + A_i)\, \phi \partial_i \phi - g\lambda_\mu A^\mu\, A^2] \tag{31}$$

$$Q^1_\mu = \phi(\pi_\phi - (\lambda_0 + A\,_0)\phi) g_{\mu 0} - \phi \partial_i \phi g_{i\mu} - \lambda_\mu A^2 - 2\lambda_\nu A^\nu A_\mu \tag{32}$$

$$Q^2_\mu = \phi(\pi_\phi - (\lambda_0 + A_0)\phi) g_{\mu 0} - \phi \partial_i \phi g_{i\mu} - A_\mu A^2 \tag{33}$$

M is an eight by eight matrix whose entities are made out of derivatives of Q^1_μ and Q^2_μ with respect to the fields A_μ and λ_μ.

We can use the integral representation of the Dirac delta functions,

$$\delta(Q^1_\mu) = \frac{1}{2\pi} \int dB_\mu \exp(-iB^\mu Q_\mu^1) \tag{34}$$

$$\delta(Q^2_\mu) = \frac{1}{2\pi} \int dE_\mu \exp(-iE^\mu Q^2_\mu) \tag{35}$$

Using ghost, i.e., Grassmann-valued, fields c_μ, e_μ, $c^\dagger_\mu$, $e^\dagger_\mu$, we can raise det M to the exponential,

$$\det M = \int dc^\dagger_\mu\, dc_\nu de^\dagger_\sigma de_\rho \exp(iN) \tag{36}$$

where

$$\begin{aligned} N = {} & (c^\dagger{}_\mu + e^\dagger_\mu)(g^2 \phi^2 g_{\mu 0} g_{\nu 0})(c_\nu + e_\nu) \\ & + c^\dagger_\mu(-2gA_\mu \lambda_\nu - 2g g_{\mu\nu} \lambda_\kappa A^\kappa - 2g\lambda_\mu\, A_\nu - 2g\lambda_\nu A\,_\mu) c_\nu \\ & + c^\dagger_\mu(-g^{\mu\nu} A^2 - 2gA^\mu A^\nu) e_\nu + e^\dagger_\mu(-g g^{\mu\nu} A^2 - 2gA^\mu A^\nu) c_\nu \end{aligned} \tag{37}$$

When the momentum integrals are performed we get

$$\begin{aligned} L_{\text{eff}} = i \Bigg[& \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + igG_\mu \phi \partial^\mu \phi - \frac{g^2}{2}(B_0 + E_0)^2\, \phi^2 - g\lambda_\mu\, A^\mu A^2 + gB^\mu \lambda_\mu\, A^2 \\ & + 2gB_\mu A^\mu \lambda_\nu A^\nu + gE\mu A_\mu A^2 + g^2 f^\dagger_0 \phi^2 f_0 + c^\dagger_\mu(-g g^{\mu\nu} A^2 - 2gA^\mu A^\nu) f_\nu \\ & + f^\dagger_\mu(-g g^{\mu\nu} A^2 - 2gA^\mu A^\nu) e_\nu \\ & + c^\dagger_\mu [2gA^2 g^{\mu\nu} - 2g\lambda^\mu A^\nu - 2g g^{\mu\nu} \lambda_\kappa A^\kappa - 2gA^\mu \lambda^\nu + 4gA^\mu A^\nu] c_\nu \end{aligned} \tag{38}$$

Here $f_\mu = c_\mu + e_\mu$. We set $G_\mu = A_\mu + \lambda_\mu - B_\mu - E_\mu$.

We perform the integration over ϕ and obtain

$$
\begin{aligned}
S_{\text{eff}} = &-\frac{1}{2}\,\text{Tr}\,\log\left[-\partial^2 + igG_\mu\partial^\mu - ig\partial_\mu G - \frac{g^2}{2}(B_0 + E_0)^2 + g^2 f_0^\dagger f_0\right] \\
&+ \int d^4x[-g\lambda^\mu A_\mu A^2 + gB_\mu\lambda^\mu A^2 + 2g\lambda^\mu A_\mu A_\nu B^\nu + gE^\mu A_\mu A^2 \\
&+ e_\mu^\dagger(-gA^2 g^{\mu\nu} - 2gA^\mu A^\nu)f_\nu \\
&+ f_\mu^\dagger(-gA^2 g^{\mu\nu} - 2gA^\mu A^\nu)e_\nu + c_\mu^\dagger(2gA^2 g^{\mu\nu} \\
&+ 4gA^\mu A^\nu - 2g\lambda^\mu A^\nu - 2g\lambda^\nu A^\mu - 2g\lambda_\rho A^\rho g^{\mu\nu})c_\nu]
\end{aligned} \tag{39}
$$

Note that only G_μ propagates among all the fields given above. To find the propagator, we take two derivatives with respect to the respective fields,

$$
\left.\frac{\partial^2 S_{\text{eff}}}{\partial G_\mu(x)\partial G_\nu(y)}\right|_0 = \frac{-1}{(2\pi)^4}\,g^2 \int d^4p\,\frac{(p^\mu + q^\mu)(p^\nu + 2q^\nu)}{p^2(p+q)^2} \tag{40}
$$

Subscript zero on the derivative means that all the fields are put to zero after the differentiation is performed.

Note that this is the same expression for the propagator of the λ_μ field as given in Eq. (26). We also see that

$$
\frac{\partial^2 S_{\text{eff}}}{\partial B_0^2} = \frac{\partial^2 S_{\text{eff}}}{\partial E_0^2} = \frac{\partial^2 S_{\text{eff}}}{\partial g_0^\dagger \partial g_0} = \frac{1}{(2\pi)^4}\int \frac{d^4p}{p^2} \tag{41}
$$

This expression is zero by dimensional regularization. All the other fields also have zero propagators since the effective Lagrangian does not have any terms which are only bilinear in these fields. All these terms involve quartic interactions of these fields. When we drop all the fields with zero propagators we end up with

$$
S''_{\text{eff}} = -\tfrac{1}{2}\,\text{Tr}\,\log(-\partial^2 - igG_\mu\partial^\mu + ig\partial_\mu G^\mu) \tag{42}
$$

This is the same expression we found for model A. Therefore all the results obtained for model A from this expression are also true for model B. We can not differentiate model A from model B as far as perturbative expansion in terms of Feynman diagrams is concerned.

4. CONCLUSION

Here we have studied two very dissimilar models which have the same Feynman expansions. A complete constrained Hamiltonian analysis shows that the two models are different. One reason we have studied this problem is to be able to clarify the behavior of many Nambu–Jona-Lasinio-like[10,3] models which are claimed to be similar to QCD.[11] Some workers[12] disagree with equivalence. The claim in ref. 12 is that after an investigation of a lattice Nambu–Jona-Lasinio model both by the Monte Carlo method and Schwinger–Dyson equations, studying renormalization group flows in the neighborhood of the critical coupling where the chiral symmetry-breaking phase transition takes place reveals no region of the bare parameter space renormalizability of the model. We propose that, in addition to the standard methods of looking at the renormalization flow and fixed-point structure of two models to show equivalence, their constrained analysis may be another check. Still another method is to study the predictions of these models for different physical processes. Some older work[5,7] seems to suggest that the Nambu–Jona-Lasinio-type models may indeed be trivial for four-dimensional space-time, perhaps like the ϕ^4 model.

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